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ON ALPHA-KENMOSU MANIFOLDS OF DİMENSİON 3 WITH CERTAIN PSEUDOSYMMETRY CONDİTİONS

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ABSTRACT

In this paper, we study some certain pseudosymmetric conditions on alpha-Kenmotsu manifolds with dimension 3. In particular, we consider Ricci generalized pseudosymmetric and Ricci pseudosymmetric manifolds. Also, we obtain some results satisfying certain curvature conditions on such manifolds depending on alpha.

KEYWORDS:

Alpha-Kenmotsu manifold; Pseudosymmetry; Ricci generalized pseudosymmetry; Ricci pseudosymmetry; Einstein manifold.

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1. INTRODUCTION

Let (M, g) be an n-dimensional $(n \geq 3)$ differentiable manifold of class C^{∞}. Denote by ∇ its Levi-Civita connection. Also, we define endomorphisms $R(X, Y)$ and $(X \wedge Y)$ by the following realtions:

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

and

$$(X \wedge Y)Z = g(Y,Z)X - g(Z,X)Y, \qquad (1.1)$$

respectively[20]. Here $X, Y, Z \in \chi(M), \chi(M)$ being the Lie aigebra of vector fields on M. The Riemannian Christoffel curvature tensor R is defined as

$$g(R(X,Y)V,W) = g(R(X,Y)V,W)(1.2)$$

where $W \in \chi(M)$.

We define the tensors $R.R$, $R.S$, $Q(g, R)$and $Q(g, S)$ by the following relations:

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$$(R(X,Y)\cdot R)(U,W)Z = R(X,Y)R(U,W)Z - R(R(X,Y)U,W)Z - R(U,R(X,Y)W)Z - R(U,W)R(X,Y)Z, \quad (1.3)$$

$$(R(X,Y)\cdot S)(U,W) = -S(R(X,Y)U,W) - S(U,R(X,Y)W, \quad (1.4)$$

$$Q(g,R)(U,W,Z;X,Y) = (X\wedge Y)R(U,W)Z - R((X\wedge Y)U,W)Z - R(U,(X\wedge Y)W)Z - R(U,W)(X\wedge Y)Z, \quad (1.5)$$

and

$$Q(g,S)(U,W;X,Y) = -S((X\wedge Y)U,W) - S(U,(X\wedge Y)W), \quad (1.6)$$

where $X,Y,Z,U,W \in \chi(M)$ [3].

If the tensors $R.R$ and $Q(g,R)$ are linearly dependent, then the manifold M is said to be pseudosymmetric. This condition is equivalent to

$$R.R = L_R Q(g,R) \quad (1.7)$$

holding on the set

$$U_R = \{x \in M: Q(g,R) \neq 0, at\ the\ point\ x\}$$

where L_R is some function on U_R[3].

If $R.R = 0$ then the manifold M is called semisymmetric. We know that every semisymmetric manifold is pseudosymmetric. But the converse is not true. Moreover, if $\nabla R = 0$ then the manifold M is said to be locally symmetric. Also, it is obvious that if the manifold M is locally symmetric then it is semisymmetric [14].

Thenotion of semi-symmetricmanifold is definedby$R(X,Y)\cdot R = 0$, for all vector fields X,Y on M, where $R(X,Y)$ acts as a derivation on R, [9]. Such a space is called "semi-symmetricspace" since thecurvaturetensor of (M,g) at a point p∈M, Rp; is the same as the curvature tensor of a symmetric space (that can change with the point of p). Thus locally symmetric spaces are obviously semi-symmetric, but the converse is not true[4].

Also, if the tensors $R.S$ and $Q(g,S)$ are linearly dependent then the manifold M is said to be Ricci pseudosymmetric. This condition is equivalent to

$$R.S = L_S Q(g,S) \quad (1.8)$$

holding on the set

$$U_S = \{x \in M: S \neq (r/n), at\ the\ point\ x\}$$

where $L_S$is some function on U_S [3].

It is wellknown that every pseudosym metric manifold is Ricci pseudosym metric but the converse is not true. If $R.S = 0$ then the manifold M is said to be Ricci semisymmetric. Every semisymmetric manifold is Ricci semisymmetric but the converse is not true. Moreover, every Ricci semisym metric manifold is also Ricci pseudosymmetric but the converse is not true [18].

Furthermore, if the tensors $R.R$ and $Q(S,R)$are linearly dependent then the manifold M is said to be Ricci generalized pseudosym metric. This condition is equivalent to

$$R.R = L\ Q(S,R) \quad (1.9)$$

holding on the set

$$U = \{x \in M: Q(S,R) \neq 0, at\ the\ point\ x\}$$

where L is some function on U [3]. Here the tensors$Q(S,R)$ ve $(X \wedge_S Y)$ are defined as

$$Q(S,R)(U,W,Z;X,Y) = (X \wedge_S Y)R(U,W)Z - R((X \wedge_S Y)U,W)Z$$
$$-R(U,(X \wedge_S Y)W)Z - R(U,W)(X \wedge_S Y)Z \quad (1.10)$$

and

$$(X \wedge_S Y)Z = S(Y,Z)X - S(X,Z)Y, \quad (1.11)$$

respectively [13].

Semisym metric Kenmotsu and alpha-Kenmotsu manifolds are studied in [13,14]. Also, the other semisymmetric conditions are investigated in [16,17,19] on such manifolds. In this paper, we study certain pseudosym metric conditions on three dimensional alpha-Kenmotsu manifolds. In particular, we obtain some results satisfying some certain curvature conditions on such manifolds depending on α.

2. RESEARCH METHOD

Let M^{2n+1} almost contact manifold be an odd-dimensional manifold. The triple (φ,ξ,η) is defined as follow. It transports a field φ of endomorphisms of the tangent spaces, ξ is a vector field that is called characteristic or Reeb vector field and η is a 1-form such that $\varphi^2 = -I + \eta \otimes \xi$ and $\eta(\xi) = 1$. The mapping defined by $I: TM^{2n+1} \to TM^{2n+1}$ is called identity mapping. By using the definition of these it follows that $\varphi\xi = 0$, $\eta \circ \varphi = 0$ and that the (1,1)-tensor field φ has constant rank $2n$ [1]. An almost contact manifold $(M^{2n+1},\varphi,\xi,\eta)$ is said to be normal if the Nijenhuis torsion tensor of φ; $N = [\varphi,\varphi] + 2d\eta \otimes \xi$ vanishes for any vector fields X, Y on M^{2n+1}. If M^{2n+1} admits a Riemannian metric g, such that

$$g(\varphi X,\varphi Y) = g(X,Y) - \eta(X)\eta(Y), \quad (2.1)$$

for any vector fields X, Y on M^{2n+1}, then this metric g is said to be a compatible metric and the manifold M^{2n+1} together with the structure $(M^{2n+1},\varphi,\xi,\eta,g)$ is called an almost contact metric manifold. Hence, (2.1) means that $\eta(X) = g(X,\xi)$ for any vector field X on M^{2n+1}. On such a manifold, the fundamental 2-form Φ of M^{2n+1} is defined by $\Phi(X,Y) = g(\varphi X,Y)$. Almost contact metric manifolds such that both η and Φ are closed are called almost cosymplectic manifolds and almost contact metric manifolds such that $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi$ are almostKenmotsu manifolds. It is noted that a normal almost cosymplectic manifold is called a cosymplectic manifold and a normal almost Kenmotsu manifolds is called Kenmotsu manifold [8,9].
An almost contact metric manifold M^{2n+1} is said to be almost alpha-Kenmotsu if $d\eta = 0$ and $d\Phi = 2\alpha\eta \wedge \Phi$, α being a non-zero real constant [8,10].
Now, we set $A = -\nabla\xi$ and $h = (1/2)(L_\xi \varphi)$. Obviously, $A(\xi) = 0$ and $h(\xi) = 0$. Moreover, the tensor fields A and h are symmetric operators and satisfy the following relations

$$\nabla_X \xi = -\alpha\varphi^2 X - \varphi h X, \quad (2.2)$$
$$(\varphi \circ h)X + (h \circ \varphi)X = 0, \quad (2.3)$$
$$(\varphi \circ A)X + (A \circ \varphi)X = -2\alpha\varphi, \quad (2.4)$$

for any vector fields X, Y on M^{2n+1}. We also remark that $h = 0 \Leftrightarrow \nabla\xi = -\alpha\varphi^2$ [14,15].
For an almost alpha-Kenmotsu manifold, the following curvature properties are held:

$$R(X,Y)\xi = (\nabla_Y \varphi h)X - (\nabla_X \varphi h)Y - \alpha[\eta(X)\varphi hY - \eta(Y)\varphi hX](Y + [\alpha^2 + \xi(\alpha)][\eta(X)Y - \eta(Y)X], \quad (2.5)$$

$$R(X,\xi)\xi = [\alpha^2 + \xi(\alpha)]\varphi^2 X + 2\alpha\varphi hX - h^2 X + \varphi(\nabla_\xi h)X, \quad (2.6)$$

$$(\nabla_\xi h)X = -\varphi R(X,\xi)\xi - [\alpha^2 + \xi(\alpha)]\varphi X - 2\alpha hX - \varphi h^2 X, (2.7)$$

$$R(X,\xi)\xi - \varphi R(\varphi X,\xi)\xi = 2[(\alpha^2 + \xi(\alpha))\varphi^2 X - h^2 X], (2.8)$$

$$S(X,\xi) = -2n[\alpha^2 + \xi(\alpha)]\eta(X) - (div(\varphi h))X, \ (2.9)$$

$$S(\xi,\xi) = -[2n(\alpha^2 + \xi(\alpha)) + tr(h^2)], (2.10)$$

for any vector fields X, Y on M^{2n+1} where α is a smooth function such that $d\alpha \wedge \eta = 0$. In these formulas, ∇ is the Levi-Civita connection and R the Riemannian curvature tensor of M^{2n+1} [16,17].

3. RESULTS AND ANALYSIS

In this section, we consider Ricci pseudosymmetric and Ricci generalized pseudosymmetric alpha-Kenmotsu manifolds with dimension 3. Here, α is a smoothfunctionsuchthat $d\alpha \wedge \eta = 0$ or a real constant.

Now, wegivethefollowinglemmasforlaterusage:

Lemma 3.1 Let M^n be an alpha-Kenmotsu manifold. Then the following equations are held:

$$(\nabla_X \varphi)Y = -\alpha[g(X,\varphi Y)\xi + \eta(Y)\varphi X], \quad (3.1)$$

$$\nabla_X \xi = -\alpha(-X + \eta(X)\xi), (\nabla_X \eta)Y = -\alpha[-g(X,Y) + \eta(X)\eta(Y)], \quad (3.2)$$

where α is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$. In special case, if $\alpha = 0$, then the manifold is cosymplectic one. Also, if $\xi(\alpha) = \nabla_\xi \alpha$ such that $(\alpha^2 + \xi(\alpha)) \neq 0$, then the alpha-Kenmotsu manifold is regular [8]. It is important to say that the condition $d\alpha \wedge \eta = 0$ satisfies for dimension is greater and equal than 5. This condition does not hold for the three dimensional case [19]. Accordingly, since the conformal curvature tensor in the three dimensional space will be identical to zero, we can also make the Riemannian curvature tensor calculations on the conformal curvature tensor. In other words, we have

$$R(X,Y)Z = S(Y,Z)X - S(Z,X)Y + g(Y,Z)QX - g(Z,X)QY - (r/2)[(X \wedge Y)Z]. \quad (3.3)$$

So we can give the following results for the three dimensional case:

Lemma3.2 In three dimensional alpha-Kenmotsumani folds, we have the following relations:

$$R(X,Y)Z = 2(\alpha^2 + \xi(\alpha) + (r/4))((X \wedge Y)Z) \quad (3.4)$$
$$-3(\alpha^2 + \xi(\alpha) + (r/6))[\eta(X)(\xi \wedge Y)Z + \eta(Y)(X \wedge \xi)Z]$$

$$S(X,Y)Z = (\alpha^2 + \xi(\alpha) + (r/2))g(X,Y) - 3(\alpha^2 + \xi(\alpha) + (r/6))\eta(X)\eta(Y), \quad (3.5)$$

where α is a strictly positive function such that $d\alpha \wedge \eta = 0$. Moreover, R, S, Q and r are the Riemannian curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature, respectively [19].

Lemma 3.3 In three dimensional alpha-Kenmotsumani folds, taking in to account of (3.3), (3.4) and (3.5), the following curvature relations are held:

$$R(X,Y)\xi = (\alpha^2 + \xi(\alpha))[\eta(X)Y - \eta(Y)X], \quad (3.6)$$

$$R(\xi,X)Y = (\alpha^2 + \xi(\alpha))[-g(X,Y)\xi + \eta(Y)X], \quad (3.7)$$

$$g(R(X,Y)Z,\xi) = (\alpha^2 + \xi(\alpha))[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \quad (3.8)$$

$$S(Y,\xi) = -2(\alpha^2 + \xi(\alpha))\eta(Y), \quad (3.9)$$

$$Q\xi = -2\left(\alpha^2 + \xi(\alpha)\right)\xi \,, \quad (3.10)$$

[14,15,17].
Thus we give the following results:

Theorem 3.1 LetM^3be an alpha-Kenmotsumani fold. If$M^3$is Ricci generalized pseudosymmetric then M^3 is an Einstein manifold such that$(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$ where α is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$.

ProofAssumethatM^3be a Ricci generalized pseudosymmetric alpha-Kenmotsumani fold. Sousing (1.9), (1.10) and (1.11) suchthat

$$(R(X,Y) \cdot R)(U,V)Z = \alpha((X \wedge_S Y) \cdot R)(U,V)Z$$

wehave

$$R(X,Y)R(U,V)Z - R(R(X,Y)U,V)Z - R(U,R(X,Y)V)Z$$
$$-R(U,V)R(X,Y)Z = \alpha[S(Y,R(U,V)Z)X - S(X,R(U,V)Z)Y$$
$$-S(Y,U)R(X,V)Z + S(X,U)R(Y,V)Z - S(Y,V)R(U,X)Z$$
$$+S(X,V)R(U,Y)Z - S(Y,Z)R(U,V)X + S(X,Z)R(U,V)Y]. \quad (3.11)$$

By the help of (3.7) and (3.9), the above equation reduces to

$$-(\alpha^2 + \xi(\alpha))[(\alpha^2 + \xi(\alpha))g(V,Z)Y + R(Y,V)Z - (\alpha^2 + \xi(\alpha))g(Y,Z)V]$$
$$= \alpha[(\alpha^2 + \xi(\alpha))\eta(Z)S(Y,V)\xi - 2(\alpha^2 + \xi(\alpha))^2 g(V,Z)Y \qquad - 2(\alpha^2 +$$
$$\xi(\alpha))R(Y,V)Z + 2(\alpha^2 + \xi(\alpha))^2\eta(V)g(Y,Z)\xi \quad (3.12)$$
$$+(\alpha^2 + \xi(\alpha))\eta(V)S(Y,Z)\xi - (\alpha^2 + \xi(\alpha))S(Y,Z)V + 2(\alpha^2 + \xi(\alpha))^2\eta(Z)g(Y,V)\xi],$$

for$U = Y = \xi$.Then the sum for$1 \leq i \leq 3$ of (3.12) for suitable contraction with respect to V and Zyields

$$0 = (3\alpha - 1)(\alpha^2 + \xi(\alpha))[S(Y,K) + 2(\alpha^2 + \xi(\alpha))g(Y,K)]$$
(3.13)

So (3.13) satisfies either$(3\alpha - 1)(\alpha^2 + \xi(\alpha)) = 0orS(Y,K) = \lambda g(Y,K)$ such that $\lambda = -2\left(\alpha^2 + \xi(\alpha)\right)$.Thus if we choose$(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$, we have desired result. This completes the proof.

Theorem 3.2 LetM^3 be an alpha-Kenmotsumani fold. IfM^3 is Ricci generalized pseudosym metric then M^3 is an Einstein manifold such that $(3\alpha - 1)\alpha^2 \neq 0$where$\alpha$ is a positive constant.

ProofAccording to the hypothesis, using the same methodology as in the above theorem we have

$$\begin{aligned} &-\alpha^2[\alpha^2 g(V,Z)g(Y,K) + g(R(Y,V)Z,K) - \alpha^2 g(Y,Z)g(V,K) \\ &= \alpha[\alpha^2\eta(Z)\eta(K)S(Y,V) - 2\alpha^4 g(V,Z)g(Y,K) - 2\alpha^2 g(R(Y,V)Z,K) \\ &\quad +2\alpha^4\eta(V)\eta(K)g(Y,Z) + \alpha^2\eta(V)\eta(K)S(Y,Z) - \alpha^2 S(Y,Z)g(V,K) \\ &\quad +2\alpha^4\eta(Z)\eta(K)g(Y,V)\xi]. \end{aligned} \quad (3.14)$$

Then the sum for$1 \leq i \leq 3$ of (3.14) for suitable contraction withr espect to$V$ and Z yields

$$0 = \alpha^2(3\alpha - 1)[S(Y,K) + 2\alpha^2 g(Y,K)].\quad(3.15)$$

This means that it holds$S(Y,K) = -2\alpha^2 g(Y,K)$ such that $\alpha^2(3\alpha - 1) \neq 0$. Here, λ is given by $\lambda = -2\alpha^2$. Thus it completes the proof.

Theorem 3.3 LetM^3 be an alpha-Kenmotsumani fold. If the condition (1.9) holds on M^3if and only i fit is locally i so metric to the hyperbolic space$H^3(-\alpha^2)$such that$(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$ andα is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$.

Proof Firstly, suppose that the manifold $M^3$is locally iso metric to the hyperbolic space$H^3(-\alpha^2)$. Then we have

$$R \cdot R = LQ(S,R) = 0. \quad (3.16)$$

[8,9]. So using the same methodology as in Theorem 3.1 by the help of (1.9) and (1.10), we get

$$\begin{aligned} &-(\alpha^2 + \xi(\alpha))[R(Y,V,Z,K) + (\alpha^2 + \xi(\alpha))g(V,Z)g(Y,K) \\ &-(\alpha^2 + \xi(\alpha))g(Y,Z)g(V,K)] \\ &= \alpha[(\alpha^2 + \xi(\alpha))\eta(Z)\eta(K)S(Y,V) - 2(\alpha^2 + \xi(\alpha))^2 g(V,Z)g(Y,K) \\ &-2(\alpha^2 + \xi(\alpha))g(R(Y,V)Z,K) + 2(\alpha^2 + \xi(\alpha))^2\eta(V)\eta(K)g(Y,Z) \\ &+(\alpha^2 + \xi(\alpha))\eta(V)\eta(K)S(Y,Z) - (\alpha^2 + \xi(\alpha))S(Y,Z)g(V,K) \\ &+2(\alpha^2 + \xi(\alpha))^2\eta(Z)\eta(K)g(Y,V)\xi] \end{aligned} \quad (3.17)$$

From (3.17) we obtain

$$S(Y,K) = -2(\alpha^2 + \xi(\alpha))g(Y,K) \quad (3.18)$$

and

$$r = S(e_i, e_i) = -6(\alpha^2 + \xi(\alpha)) \tag{3.19}$$

where $(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$. Using (3.18) in (1.9) we have

$$R \cdot R = -2(\alpha^2 + \xi(\alpha))Q(g, R) . \tag{3.20}$$

On the other hand, we know that every alpha-Kenmotsumani foldM^3 is a pseudosym metric manifold of the form

$$R \cdot R = -(\alpha^2 + \xi(\alpha))Q(g, R) \tag{3.21}$$

where α is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$.So this gives a contradiction. Hence, $R \cdot R = 0$ holds. Since the manifold M^3 is semisym metric, it is locally iso metric to the hyperbolic space$H^3(-\alpha^2)$ [8,9]. This completes the proof.

Theorem 3.4 LetM^3 be an alpha-Kenmotsumani fold. If the condition (1.9) holds on M^3if and only if it is locally iso metric to the hyperbolic space$H^3(-\alpha^2)$ such that $\alpha^2(3\alpha - 1) \neq 0and\alpha$ is positive constant.

Proof Analogously, considering the above theorem for positive constant α then we have

$$S(Y, K) = -2\alpha^2 g(Y, K) \tag{3.22}$$

and

$$r = S(e_i, e_i) = -6\alpha^2 \tag{3.23}$$

where$\alpha^2(3\alpha - 1) \neq 0$.Thenusing (3.15) in (1.9) weobtain

$$R \cdot R = -2\alpha^2 Q(g, R)$$

And also we note that (3.21)holds for positive constant α. Hence, there is a contradiction. Thus it completes the proof.

Theorem 3.5 LetM^3 be a Ricci generalized pseudosym metric alpha-Kenmotsumani fold. If the manifold M^3 is not semi symmetric then it is an Einstein manifold satisfying$r = -6(\alpha^2 + \xi(\alpha))$ and $L = (1/2)$sucht hat$(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$. Here α is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$.

Proof Assume that M^3 is Ricci generalized pseudosym metric alpha-Kenmotsumani fold. Using the same method as in Theorem 3.1 we have the following relation:

$$\begin{aligned} &-(\alpha^2 + \xi(\alpha))R(Y, V, Z, K) - (\alpha^2 + \xi(\alpha))^2 g(V, Z)g(Y, K) \\ &\quad + (\alpha^2 + \xi(\alpha))^2 g(Y, Z)g(V, K) \\ &= L\{\alpha[(\alpha^2 + \xi(\alpha))\eta(Z)\eta(K)S(Y, V) - 2(\alpha^2 + \xi(\alpha))^2 g(V, Z)g(Y, K) \\ &-2(\alpha^2 + \xi(\alpha))g(R(Y, V)Z, K) + 2(\alpha^2 + \xi(\alpha))^2 \eta(V)\eta(K)g(Y, Z) \\ &+(\alpha^2 + \xi(\alpha))\eta(V)\eta(K)S(Y, Z) - (\alpha^2 + \xi(\alpha))S(Y, Z)g(V, K) \\ &\quad +2(\alpha^2 + \xi(\alpha))^2 \eta(Z)\eta(K)g(Y, V)\xi]\}. \end{aligned} \tag{3.24}$$

Then (3.24) reduces to

$$0 = (3\alpha - 1)(\alpha^2 + \xi(\alpha))L[S(Y,K) + 2(\alpha^2 + \xi(\alpha))g(Y,K)]. \quad (3.25)$$

Since M^3 is not semisym metric, it holds$L \neq 0$. Thus (3.25) is of the form

$$S(Y,K) = -2(\alpha^2 + \xi(\alpha))g(Y,K)$$

such that $(3\alpha - 1)(\alpha^2 + \xi(\alpha)) \neq 0$.There fore, the manifold M^3 is an Einstein manifold with $r = -6(\alpha^2 + \xi(\alpha))$. Taking into account of (1.9) and the above equation we have

$$R \cdot R = -2\left(\alpha^2 + \xi(\alpha)\right)LQ(g,R). \quad (3.26)$$

But it is noted that

$$-2\left(\alpha^2 + \xi(\alpha)\right)L = -\left(\alpha^2 + \xi(\alpha)\right).$$

So this leads to $L = (1/2)$.Thisprovestotheorem.

Theorem 3.6 LetM^3 be a Ricci generalized pseudosym metric alpha-Kenmotsu manifold. If the manifold M^3 is not semi symmetric then it is an Einstein mani fold satisfying $r = -6\alpha^2$and$L = (1/2)$ such that $(3\alpha - 1)\alpha^2 \neq 0$ where α is positive constant.

Proof Suppose that $M^3$is Ricci generalized pseudosym metric alpha-Kenmotsu manifold.By the help of Theorem 3.1, we have

$$\begin{aligned}
&-\alpha^2 R(Y,V,Z,K) - \alpha^4 g(V,Z)g(Y,K) + \alpha^4 g(Y,Z)g(V,K) \\
&= L\{\alpha[\alpha^2\eta(Z)\eta(K)S(Y,V) - 2\alpha^4 g(V,Z)g(Y,K) \\
&-2\alpha^2 g(R(Y,V)Z,K) + 2\alpha^4\eta(V)\eta(K)g(Y,Z) \\
&+\alpha^2\eta(V)\eta(K)S(Y,Z) - \alpha^2 S(Y,Z)g(V,K) \quad (3.27) \\
&+2\alpha^4\eta(Z)\eta(K)g(Y,V)\xi]\}.
\end{aligned}$$

Then (3.27) reduces to

$$0 = (3\alpha - 1)\alpha^2 L[S(Y,K) + 2\alpha^2 g(Y,K)] \quad (3.28)$$

Here, using the same operations as in the above theorem, the proof is obvious from (3.28) for $\alpha^2(3\alpha - 1) \neq 0$.

Theorem 3.7 IfM^3 be a Ricci pseudosym metric alpha-Kenmotsumani fold then it is an Eistein manifold given by$r = -6(\alpha^2 + \xi(\alpha))$ when the first and third of the selected arbitrary vector fields restricted to ξ vector field such that $L_S \neq -\left(\alpha^2 + \xi(\alpha)\right)$.

ProofFirst we suppose that M^3 is Ricci pseudosym metric alpha-Kenmotsu manifold. Then w ehave

$$(R(X,Y)\cdot S)(U,W) = L_S Q(g,S)(U,W;X,Y) \quad (3.29)$$

for arbitrary vector fields on M^3. From (3.29), we also get

$$(R(X,Y)\cdot S)(U,W) = L_S((X\wedge_g Y)\cdot S)(U,W) \quad (3.30)$$

or

$$L_S[-g(U,Y)S(X,W) + g(X,U)S(Y,W) - g(Y,W)S(U,X) \\ +g(X,W)S(U,Y)] = -S(R(X,Y)U,W) - S(U,R(X,Y)W). \quad (3.31)$$

Now, if we put the first and third of the selected arbitrary vector fields restricted to ξ, that is, taking $X = U = \xi$, then (3.31) turns into

$$L_S[-\eta(Y)S(\xi,W) + S(Y,W) - g(Y,W)S(\xi,X) \\ +\eta(W)S(U,Y)] = -S(R(\xi,Y)\xi,W) - S(\xi,R(\xi,Y)W) (3.32)$$

with the help of (3.7) and (3.9). Here we take $K = -(\alpha^2 + \xi(\alpha))$, we have

$$KS(Y,W) - 2K^2 g(Y,W) = L_S[S(Y,W) - 2Kg(Y,W)]. \quad (3.33)$$

Simplifying the last equation, we obtain

$$0 = [K - L_S][S(Y,W) - 2Kg(Y,W)] \quad (3.34)$$

or

$$0 = \left[L_S + \left(\alpha^2 + \xi(\alpha)\right)\right]\left[S(Y,W) + 2\left(\alpha^2 + \xi(\alpha)\right)g(Y,W)\right]. \quad (3.35)$$

So under these restrictions for $L_S \neq -(\alpha^2 + \xi(\alpha))$ we have

$$S(Y,W) = \lambda g(Y,W)$$

with $\lambda = -2(\alpha^2 + \xi(\alpha))$. It is clear that the scalar curvature of the manifold is defined by $r = -6(\alpha^2 + \xi(\alpha))$. Thus it completes the proof.

Corollary 3.1 Let M^3 be an alpha-Kenmotsu manifold. If M^3 is Ricci pseudosym metric, then either it holds $L_S = -(\alpha^2 + \xi(\alpha))$ or the manifold is an Einstein given by $r = -6(\alpha^2 + \xi(\alpha))$ where α is strictly positive function of class C^∞ such that $d\alpha \wedge \eta = 0$.

Corollary 3.2 Let M^3 be an alpha-Kenmotsu manifold. If M^3 is Ricci pseudosym metric, then either it holds $L_S = -\alpha^2$ or the manifold is an Einstein given by $r = -6\alpha^2$ where α is a positive constant.

4. CONCLUSION

This paper deals with alpha-Kenmotsu manifolds satisfying certain pseudosym metry conditions for 3-dimensional case. Some certain results are obtained related tocurvaturetensors on suchmanifolds. Our forthcoming paper is devoted to investigate three dimensional almost alpha-Kenmotsu manifolds satisfying the other pseudosymmetry conditions. There are quite open problems waiting to be proved.

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